Functions

Math 130 - Essentials of Calculus

9 September 2019

Combining Functions

Another way to combine functions is by composition.

Definition

Given two functions f and g, the composition of f and g is defined by

$$h(x) = f(g(x)).$$

Essentially, this is just putting the output of g in as an input of f.

Example

Let $f(x) = x^2 + 1$ and $g(t) = 4t - 2$. Let $h(t) = f(g(t))$ and $k(x) = g(f(x))$.

1. *Compute $h(3)$, then find a formula for $h(t)$.*
2. *Compute $k(3)$, then find a formula for $k(x)$.*

Decomposing Functions

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Example

Find functions f and g such that $h(x) = f(g(x))$.

1. $h(x) = \sqrt{x^3 - 1}$
2. $h(x) = \dfrac{1}{x^2 - 5}$

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3. $y = f(cx)$ stretches the graph horizontally by a factor of c
4. $y = f(\frac{1}{c}x)$ compresses the graph horizontally by a factor of c

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2. $y = f(-x)$ reflects the graph of $y = f(x)$ about the y-axis

Graphing Transformations

Example

Sketch a graph of $f(x) = x^2$. Then graph the following transformations of $f(x)$.

1. $y = 2f(x)$
2. $y = f(x) - 3$
3. $y = \frac{1}{4}f(x) + 2$

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More concretely, if a line passes though the points (x_1, y_1) and (x_2, y_2), then the slope of the line is given by

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Given the slope of a line, m, and a point it passes though (x_1, y_1), an equation for the line is

$$y - y_1 = m(x - x_1).$$

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2. $(2, -3)$ *and* $(-3, -5)$

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Example

The weekly ratings, in millions of viewers, of a recent television program are given by $L(w)$, where w is the number of weeks since the show premiered. If L is a linear function where $L(8) = 5.32$ and $L(12) = 8.36$, compute the slope of L and explain what it represents in this context. Write a formula for $L(w)$.

Now You Try It!

Example

The monthly cost of driving a car depends on the number of miles driven. Lynn found that in May is cost her \$380 *to drive* 480 *miles and in June it cost her* \$460 *to drive* 800 *miles.*

1. *Express the monthly cost C as a function of the distance driven d, assuming that there is a linear relationship.*
2. *Use part 1 to predict the cost of driving* 1500 *miles per month.*
3. *What does the slope represent?*